C4 Exercise 6B (integration using reverse chain rule)

Note Title 04/07/2007

MAIN RESULT:

$$\int f'(ax+b)\,dx = \frac{1}{a} f(ax+b) + C$$

COROLLARY:

10 $$\int (ax+b)^n\,dx = \frac{1}{a}\,\frac{(ax+b)^{n+1}}{n+1} + C$$

11 $$\int e^{ax+b}\,dx = \frac{1}{a}\,e^{ax+b} + C$$

12 $$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C$$

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RULES CTD.

13 $$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

14 $$\int \sin(ax+b)\,dx = \frac{-1}{a}\sin(ax+b) + C$$

15 $$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

16 $$\int \operatorname{cosec}(ax+b)\cot(ax+b)\,dx = \frac{-1}{a}\operatorname{cosec}(ax+b) + C$$

17 $$\int \operatorname{cosec}^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + C$$

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18 $\int \sec(ax+b)\tan(ax+b)\,dx = \frac{1}{a}\sec(ax+b) + C$

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1 Integrate

a) $\sin(2x+1)$

$$\int \sin(2x+1)\,dx = -\frac{1}{2}\cos(2x+1) + C$$

b) $3e^{2x}$

$$\int 3e^{2x}\,dx = \frac{3}{2}e^{2x} + C$$

c) $4e^{x+5}$

$$\int 4e^{x+5} = 4e^{x+5} + C$$

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1d $\cos(1-2x)$

$$\int \cos(1-2x)\,dx = -\frac{1}{2}\sin(1-2x) + C$$

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1e $\int \operatorname{cosec}^2 3x \, dx = -\frac{1}{3}\cot 3x + C$

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1f $\int \sec 4x \tan 4x \, dx = \frac{1}{4}\sec 4x + C$

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1g $\int 3\sin\left(\tfrac{1}{2}x+1\right)\,dx = -\frac{3}{1/2}\cos\left(\tfrac{1}{2}x+1\right) + C$

$= -6\cos\left(\tfrac{1}{2}x+1\right) + C$

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1h $\int \sec^2(2-x)\,dx = -\tan(2-x) + C$

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1i $\int \operatorname{cosec} 2x \cot 2x \, dx = -\frac{1}{2} \operatorname{cosec} 2x + C$

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1j $\int \cos 3x - \sin 3x \, dx = \frac{1}{3}\sin 3x + \frac{1}{3}\cos 3x + C$

$= \frac{1}{3}\left(\sin 3x + \cos 3x\right) + C$

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2a $\int \left(e^{2x} - \frac{1}{2}\sin(2x-1)\right) dx$

$$\int e^{2x} = \frac{1}{2}e^{2x} + C_1$$

$$\int -\frac{1}{2}\sin(2x-1)\,dx = \frac{1}{4}\cos(2x-1) + C_2$$

$$\Rightarrow \int \left(e^{2x} - \frac{1}{2}\sin(2x-1)\right)dx = \frac{1}{2}e^{2x} + \frac{1}{4}\cos(2x-1) + C$$

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2b $\int (e^x + 1)^2 \, dx = \int (e^{2x} + 2e^x + 1) \, dx$

$= \frac{1}{2}e^{2x} + 2e^x + x + C$

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2c $\int \sec^2 2x\,(1 + \sin 2x)\,dx = \int \left(\sec^2 2x + \frac{\sin 2x}{\cos 2x} \times \frac{1}{\cos 2x}\right) dx$

$$= \int \left(\sec^2 2x + \sec 2x \tan 2x\right) dx$$

$$= \frac{1}{2}\tan 2x + \frac{1}{2}\sec 2x + C$$

$$= \frac{1}{2}\left(\tan 2x + \sec 2x\right) + C$$

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2d $\displaystyle\int \frac{3 - 2\cos(\frac{1}{2}x)}{\sin^2(\frac{1}{2}x)}\,dx = \int \left[3\,\text{cosec}^2(\tfrac{1}{2}x) - 2\,\text{cosec}(\tfrac{1}{2}x)\cot(\tfrac{1}{2}x)\right]dx$

$$= -6\cot(\tfrac{1}{2}x) + 4\,\text{cosec}(\tfrac{1}{2}x) + C$$

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2e $\int \left[e^{3-x} + \sin(3-x) + \cos(3-x) \right] dx$

$$\int e^{3-x}\, dx = -e^{3-x} + C_1$$

$$\int \sin(3-x)\, dx = \cos(3-x) + C_2$$

$$\int \cos(3-x)\, dx = -\sin(3-x) + C_3$$

$$\Rightarrow \int \left[e^{3-x} + \sin(3-x) + \cos(3-x) \right] dx = -e^{3-x} + \cos(3-x) - \sin(3-x) + C$$

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3a $\int \frac{1}{2x+1}\,dx = \frac{1}{2}\ln|2x+1| + C$

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3b $\int \frac{1}{(2x+1)^2}\,dx = \int (2x+1)^{-2}\,dx$

$= \frac{1}{-1}\,\frac{1}{2}\,(2x+1)^{-1} + C$

$= \frac{-1}{4x+2} + C$ (oe)

C4 Ex 6B

3c $\int (2x+1)^2 \, dx = \frac{1}{2\times 3}(2x+1)^3 + C$

$= \frac{1}{6}(2x+1)^3 + C$

C4 Ex 6B

3d $\displaystyle\int \frac{3}{4x-1}\,dx = \frac{3}{4}\ln|4x-1| + C$

C4Ex6B

3e $\int \frac{3}{1-4x}\,dx = \frac{3}{-4}\ln|1-4x| + C$

$= \frac{-3}{4}\ln|1-4x| + C$

C4Ex6B

3f $\displaystyle\int \frac{3}{(1-4x)^2}\,dx = \int 3(1-4x)^{-2}\,dx$

$$= \frac{3}{-4}(1-4x)^{-1} + C$$

$$= \frac{3}{4(1-4x)} + C$$

$$= \frac{3}{4-16x} + C \qquad \text{(oe)}$$

C4 Ex 6B

3g $\int (3x+2)^5\,dx = \frac{1}{3} \times \frac{1}{6} \times (3x+2)^6 + C$

$= \frac{(3x+2)^6}{18} + C$

C4 Ex 6E

3h

$$\int \frac{3}{(1-2x)^3}\,dx = \int 3(1-2x)^{-3}\,dx$$

$$= \frac{1}{-2} \times \frac{3}{-2}(1-2x)^{-2} + C$$

$$= \frac{3}{4(1-2x)^2} + C$$

C4Ex6B

3i $\displaystyle\int \frac{6}{(3-2x)^4}\,dx = \int 6(3-2x)^{-4}\,dx$

$= \dfrac{6}{-2\times-3}(3-2x)^{-3} + C$

$= (3-2x)^{-3} + C$

$= \dfrac{1}{(3-2x)^3} + C$

C4 Ex6B

3j
$$\int \frac{5}{3-2x}\,dx = \frac{5}{-2}\ln|3-2x| + C$$
$$= -\frac{5}{2}\ln|3-2x| + C$$

C4 Ex6B

4a $\int \left(3\sin(2x+1) + \frac{4}{2x+1}\right) dx$

$\int 3\sin(2x+1)\,dx = -\frac{3}{2}\cos(2x+1) + C_1$

$\int \frac{4}{2x+1}\,dx = \frac{4}{2}\ln|2x+1| + C_2$

$\Rightarrow \int \left[3\sin(2x+1) + \frac{4}{2x+1}\right] dx = -\frac{3}{2}\cos(2x+1) + 2\ln|2x+1| + C$

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4b $\int \left[e^{5x} + (1-x)^5 \right] dx$

$$\int e^{5x}\,dx = \frac{1}{5}e^{5x} + C$$

$$\int (1-x)^5\,dx = -\frac{1}{6}(1-x)^6 + C'$$

$$\Rightarrow \int \left[e^{5x} + (1-x)^5 \right] dx = \frac{1}{5}e^{5x} - \frac{1}{6}(1-x)^6 + C$$

C4 Ex6B

4c $\int \left[\frac{1}{\sin^2 2x} + \frac{1}{1+2x} + \frac{1}{(1+2x)^2} \right] dx$

$$\int \frac{1}{\sin^2 2x}\,dx = \int \operatorname{cosec}^2 2x\,dx = -\frac{1}{2}\cot 2x + C_1$$

$$\int \frac{1}{1+2x}\,dx = \frac{1}{2}\ln|1+2x| + C_2$$

$$\int (1+2x)^{-2}\,dx = \frac{-1}{2}(1+2x)^{-1} + C_3$$

$$\Rightarrow \int \left[\frac{1}{\sin^2 2x} + \frac{1}{1+2x} + \frac{1}{(1+2x)^2} \right] dx = \frac{1}{2}\left[\ln|1+2x| - \cot 2x - \frac{1}{1+2x} \right] + C$$

(oe)

C4 Ex6B

4d $\int \left[(3x+2)^2 + \frac{1}{(3x+2)^2} \right] dx$

$= \frac{1}{3} \times \frac{1}{3} (3x+2)^3 + \frac{1}{3} \times -1 (3x+2)^{-1} + C$

$= \frac{(3x+2)^3}{9} - \frac{1}{3(3x+2)} + C$